On the Possibility of a Back Door in the NIST SP800-90 Dual Ec Prng

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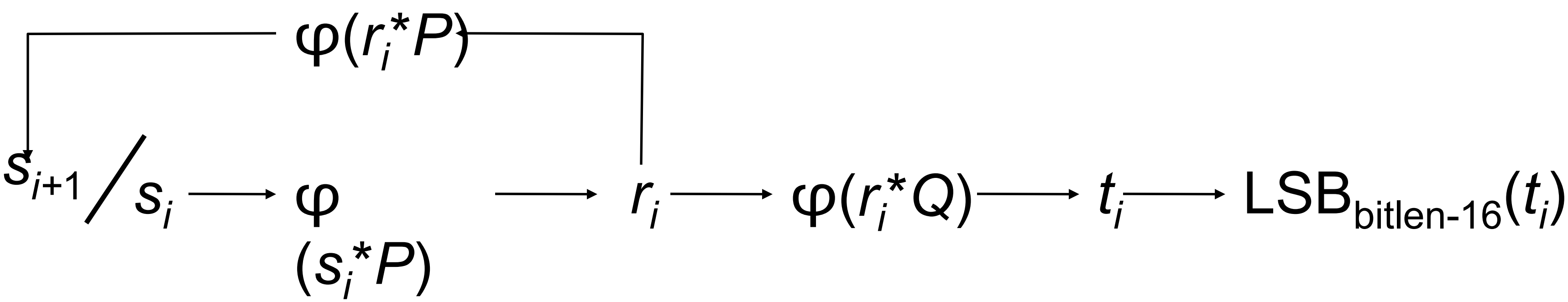
The Dual Ec PRNG

- φ : prime curve → integers
  $\varphi(x,y) = x$
- P, Q points on the curve (per SP800-90)



Equations:

$r_i = \varphi(s_i{*}P)$ $t_i = \varphi(r_i{*}Q)$ $s_{i+1} = \varphi(r_i{*}P)$

The Objection

- Point *P* is generator of the curve (per SP800-90).
- Point *Q* is a specified constant. It is not stated how it was derived.
- NIST prime curves have prime order. So there exists *e* such that $Q^e = P$.

The Attack

- Output: S, the set of possible values of s_{i+1} the internal state of the Dual Ec PRNG at the subsequent step.
- Suppose an attacker knows value e.

Given: a block of output o_i from a Dual EC PRNG Instance

Set $S = \{\}$.

For $0 \leq u \leq 2^{16} - 1$

$x = u|o_i$

$z \equiv x^3 + ax + b \bmod p$.

If $y \equiv z^{1/2} \bmod p$ exists => $A = (x,y)$ is on the curve

$S = S \cup \{\varphi(e^*A)\}$.

How this works:

- One of the values $x = t_i$

  If A is the point with x coordinate t_i then:

  $$A = r_i * Q$$

  Thus:

  $$\varphi(e*A) = \varphi(e* r_i * Q) = \varphi(r_i * P) = s_{i+1}.$$

  => s_{i+1} is in S.

- $|S| \approx 2^{15}$

Experimental Verification

1. Pick NIST P-256 Curve
2. Chose random *d*
3. Chose $Q_2 = d^*P$
4. Replace Q with Q_2
5. Given |Output| = 32 > out block length
6. Filter out s_{i+1} values that do not generate next 2 bytes.

In every experiment 32 bytes of output was sufficient to uniquely identify the internal state of the PRNG.

The Main Point

- If an attacker knows d such that $d*P = Q$ then they can easily compute e such that $e*Q = P$ (invert mod group order)
- If an attacker knows e then they can determine a small number of possibilities for the internal state of the Dual Ec PRNG and predict future outputs.
- We do not know how the point Q was chosen, so we don't know if the algorithm designer knows d or e.

Conclusion

- WHAT WE ARE NOT SAYING:
  NIST intentionally put a back door in this PRNG
- WHAT WE ARE SAYING:
  The prediction resistance of this PRNG (as presented in NIST SP800-90) is dependent on solving one instance of the elliptic curve discrete log problem.
  (And we do not know if the algorithm designer knew this before hand.)

Suggestions for Improvement

- Truncate off more than the top 16 bits of the output block.
  - Results on extractors from *x* coordinates of EC points of prime curves suggest truncating off the top bitlen/2 bits is reasonable.
- Generate a random point *Q* for each instance of the PRNG.