

Optimal Sieving Regions for the Lattice Sieve

Let

$$L = \begin{bmatrix} p & r \\ 0 & 1 \end{bmatrix} \text{ with } r = u[1, p - 1].$$

Let

$$L_{red} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

with

$$x_{11}x_{22} - x_{12}x_{21} = p.$$

Query:

What, if anything is known about the density function for x_{ij} ?

In particular, What is $E[x_{ij}]$?

What about the condition number of L_{red} ?

The norms that we want to be smooth depend on V_1 and V_2 .

Let $G(a, b, p)$ be the probability that the (norms of the) lattice point $aV_1 + bV_2$ are smooth.

Then we want to minimize

$$\int_{p=q_1}^{p=q_2} \Omega^2(p) dp$$

Subject to

$$\int_{q_1}^{q_2} \int_0^{\Omega(p)} \int_0^{\Omega(p)} G(a, b, p) da db dp > T$$